Variable selection in the presence of missing data:resampling and imputation

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1 Introduction

In the presence of missing data, variable selection methods need to be tailored to missing data mechanisms and statistical approaches used for handling missing data. We focus on the mechanism of missing at random and variable selection methods that can be combined with imputation. We investigate a general resampling approach (BISS) that combines bootstrap imputation and stability selection, the latter of which was developed for fully observed data. The proposed approach is general and can be applied to a wide range of settings. Our extensive simulation studies demonstrate that the performance of BI-SS is the best or close to the best and is relatively insensitive to tuning parameter values in terms of variable selection, compared with several existing methods for both low-dimensional and high-dimensional problems.

In this document, we will give a short tutorial on using the functions in this package to conduct imputations, variable selection, and regression modeling. BISS is the main function. This function first performs the boostrap imputation based on mice method(mice package) and balasso method(MIHD) method on data consists of outcome parameter and predictors with missing values to obtain bootstrap imputed dataset. Then this function use lasso and stability selection with randomized lasso to conduct varaible selection on the bootstrap imputated datasets, and obtain selection indicators. Lastly, this function calculate selected predictor estimates.
Function named detect.missing runs NA search and replace on the input matrix,and return a binary matrix of the same size, which can be used to detect the missing data pattern.

We give a simple example of how to view the data containing the missing values in this package and run BISS functions under different regression family options and differet imputation options.

2 Example

In this package, three datasets ("*gaussianData*","*binaryData*","*poissonData*" ,"*gaussianGeneralizedData*","*gaussianHighDimensionData*") are included. We first load our datasets.

```
> library(BISSpkg)
> data(gaussianData)
> data(binaryData)
> data(poissonData)
> data(gaussianGeneralizedData)
> data(gaussianHighDimensionData)
```

2.1 Gaussian data

First, let's take a look at dataset named gaussianData, whose type of missing values is Gaussian.

```
> dim(gaussianData)
```

```
[1] 250  11
```

```
> gaussianData[1:20,1:6]
```

```
             Y          X1          X2          X3          X4          X5
1   1.66910439          NA -0.38239954  2.23987455  0.58207947  1.03022710
2  -5.89752124  0.98179734 -0.68623953 -1.42028968 -1.31297962 -0.79433605
3   4.71138328          NA  0.87064815  0.40136174 -0.21534102  0.84962787
4   2.55691506          NA  0.96470203  1.30535720 -0.54739009  0.93955738
5   7.35728923          NA -0.45778682  0.72868730  1.14481075  2.04716810
6  -0.08226363 -0.39688067  1.23534424  0.52483873  0.36827968 -0.77277322
7   2.39109923          NA  0.04475838 -0.30954230  0.44060482  0.54967377
8   0.41359444          NA  0.98085239  0.80662866  0.64399419 -0.41387184
9   1.11792657          NA -1.82916412 -1.24108147  0.15798779  0.45819317
10 -3.57534553 -0.79543141 -3.01530412 -2.31938361  1.29179688  1.45635020
11  3.26043252          NA  0.18420068  1.01051574  0.28663553 -0.23822022
12  3.19407011          NA  0.27571273  1.15433804  2.32489829  1.80216773
13  1.30643892  0.02925003 -0.05280988  0.18911768  0.03260646  0.31169122
14  3.45004030          NA  0.75189219 -1.72502506 -0.25553791 -0.16572650
15  2.03387465          NA -0.24029326  0.05384884 -0.71032932  1.20298066
16  3.36062153          NA  0.66078562  0.45584071 -0.12771327 -0.23736481
17 -6.54895197 -1.84318579 -1.78648951 -0.96116580  0.99945948 -0.39015465
18 -0.05999473 -0.04642971  0.03410440 -0.43048817 -0.05497932 -0.64927149
19 -2.52532478  0.53899087  0.16824073  1.53860155  0.42827949  0.65846049
20  1.91503010          NA  0.57803965  0.46544429 -0.24895439 -0.06211272
```

Each column represents a variable and each row stands for a subject. And *gaussianData* has 100 subjects and 1001 variables, thus can be viewed as a high-dimensional data.

```
> sum(is.na(gaussianData))
```

```
[1] 107
```

```
> sum(is.na(gaussianData))/dim(gaussianData)[1]

[1] 0.428
```

42% of them are missing their first column values. With the help of two functions in this packages, we could impute these missing values.

First, we can use the method through direct use of bootstrap imputation and variable selection, i.e. *BISS*.

Since the variable that has missingness is Gaussian outcome, option `family="gaussian"` is used to specify the certain regressionfamily type. In this case, we use bootstrap.size of 10, means we want to get 10 bootstrap imputation sets. Here, `missing.col=1` means we spefiy the first column of the data is missing.

```
> lr_output=BI_SS(gaussianData[,-1],gaussianData$Y,family="gaussian",link=NULL,
+                 missing_col=1,MI.method="mice",BI.size=10,pi=NULL,nsteps=12)
> lr_output$BIMP[1,1:20,1:6]

             [,1]        [,2]       [,3]        [,4]       [,5]       [,6]
 [1,] -0.63448363 -0.38239954  2.2398746  0.58207947  1.0302271  1.3144419
 [2,] -0.64944297 -0.38239954  2.2398746  0.58207947  1.0302271  1.3144419
 [3,]  0.98179734 -0.68623953 -1.4202897 -1.31297962 -0.7943360 -1.4401720
 [4,]  0.04430021  0.96470203  1.3053572 -0.54739009  0.9395574  0.9576999
 [5,]  0.88643412  0.96470203  1.3053572 -0.54739009  0.9395574  0.9576999
 [6,] -0.20629018 -0.45778682  0.7286873  1.14481075  2.0471681  0.7722505
 [7,] -0.64944297  0.98085239  0.8066287  0.64399419 -0.4138718  0.8382334
 [8,] -0.64629355  0.98085239  0.8066287  0.64399419 -0.4138718  0.8382334
 [9,] -0.05683091 -1.82916412 -1.2410815  0.15798779  0.4581932  1.0159846
[10,] -0.37054307 -1.82916412 -1.2410815  0.15798779  0.4581932  1.0159846
[11,]  0.05237242  0.18420068  1.0105157  0.28663553 -0.2382202  0.7197105
[12,] -0.39692086  0.18420068  1.0105157  0.28663553 -0.2382202  0.7197105
[13,]  0.02925003 -0.05280988  0.1891177  0.03260646  0.3116912  0.6570174
[14,]  0.02925003 -0.05280988  0.1891177  0.03260646  0.3116912  0.6570174
[15,]  0.02925003 -0.05280988  0.1891177  0.03260646  0.3116912  0.6570174
[16,]  0.54523574  0.66078562  0.4558407 -0.12771327 -0.2373648  0.3308828
[17,] -0.28484748  0.66078562  0.4558407 -0.12771327 -0.2373648  0.3308828
[18,] -0.04642971  0.03410440 -0.4304882 -0.05497932 -0.6492715  0.4312311
[19,]  0.53899087  0.16824073  1.5386015  0.42827949  0.6584605 -2.0010456
[20,]  0.53899087  0.16824073  1.5386015  0.42827949  0.6584605 -2.0010456
```

```
> lr_output$S.FIN

     [,1] [,2] [,3] [,4] [,5] [,6] [,7] [,8] [,9] [,10]
 [1,]    1    1    1    1    1    1    1    1    1     1
 [2,]    1    1    0    1    1    1    1    0    1     1
 [3,]    1    1    0    0    1    1    1    0    0     1
 [4,]    1    1    1    0    1    1    1    0    1     1
 [5,]    1    1    1    0    1    1    1    0    1     1
 [6,]    1    1    1    0    1    1    1    0    1     1
 [7,]    1    1    0    0    1    1    1    0    1     1
 [8,]    1    1    0    0    1    1    1    0    1     1
```

```
 [9,]    1    1    1    1    1    1    1    1    1    1
[10,]    1    1    1    1    1    1    1    1    1    1
[11,]    1    1    1    1    1    1    1    1    1    1
[12,]    1    1    1    1    1    1    1    1    1    1
[13,]    1    1    1    1    1    1    1    1    1    1
[14,]    1    1    1    0    1    1    1    0    1    1
[15,]    1    1    1    0    1    1    1    0    1    1
[16,]    1    1    1    0    1    1    1    0    1    1
[17,]    1    1    0    0    1    1    1    0    1    1
[18,]    1    1    0    0    1    1    1    0    1    1
[19,]    1    1    1    1    1    1    1    1    1    1
[20,]    1    1    1    1    1    1    1    1    1    1
[21,]    1    1    1    1    1    1    1    1    1    1
[22,]    1    1    1    1    1    1    1    1    1    1
[23,]    1    1    1    1    1    1    1    1    1    1
```

> lr_output$BETA.FIN

```
             [,1]       [,2]     [,3]        [,4]       [,5]      [,6]      [,7]
 [1,] -0.17214234 1.0179205 1.156373 -0.03875949 0.04621452 0.8511104 0.9269521
 [2,] -0.25478925 0.9699293 1.115443  0.00000000 0.02984673 0.8093345 0.9030628
 [3,] -0.21689744 0.9982139 1.129195  0.00000000 0.00000000 0.8466663 0.9146423
 [4,]  0.05307703 1.0741582 1.214313 -0.03178187 0.00000000 0.8988740 0.9274242
 [5,]  0.05307703 1.0741582 1.214313 -0.03178187 0.00000000 0.8988740 0.9274242
 [6,]  0.05307703 1.0741582 1.214313 -0.03178187 0.00000000 0.8988740 0.9274242
 [7,]  0.05307703 1.0741582 1.214313  0.00000000 0.00000000 0.8988740 0.9274242
 [8,]  0.05307703 1.0741582 1.214313  0.00000000 0.00000000 0.8988740 0.9274242
 [9,]  0.05307703 1.0741582 1.214313 -0.03178187 0.00000000 0.8988740 0.9274242
[10,]  0.05307703 1.0741582 1.214313 -0.03178187 0.00000000 0.8988740 0.9274242
[11,]  0.05307703 1.0741582 1.214313 -0.03178187 0.00000000 0.8988740 0.9274242
[12,]  0.05307703 1.0741582 1.214313  0.00000000 0.00000000 0.8988740 0.9274242
[13,]  0.05307703 1.0741582 1.214313  0.00000000 0.00000000 0.8988740 0.9274242
[14,]  0.06269142 1.0996169 1.235842 -0.05722158 0.00000000 0.9183376 0.9361875
[15,]  0.06269142 1.0996169 1.235842 -0.05722158 0.00000000 0.9183376 0.9361875
[16,]  0.06269142 1.0996169 1.235842 -0.05722158 0.00000000 0.9183376 0.9361875
[17,]  0.05841397 1.0969546 1.208421  0.00000000 0.00000000 0.9113025 0.9314700
[18,]  0.05841397 1.0969546 1.208421  0.00000000 0.00000000 0.9113025 0.9314700
[19,]  0.05842185 1.1023103 1.234253 -0.06047287 0.01944689 0.9094509 0.9323530
[20,]  0.05842185 1.1023103 1.234253 -0.06047287 0.01944689 0.9094509 0.9323530
[21,]  0.05842185 1.1023103 1.234253 -0.06047287 0.01944689 0.9094509 0.9323530
[22,]  0.05842185 1.1023103 1.234253 -0.06047287 0.01944689 0.9094509 0.9323530
[23,]  0.05842185 1.1023103 1.234253 -0.06047287 0.01944689 0.9094509 0.9323530
         [,8]        [,9]      [,10]     [,11]
 [1,] 1.084487 -0.08409667 0.13572511 0.9057769
 [2,] 1.013321  0.00000000 0.06708279 0.8814668
 [3,] 1.043233  0.00000000 0.00000000 0.9381312
 [4,] 1.151819  0.00000000 0.21191299 0.9426468
 [5,] 1.151819  0.00000000 0.21191299 0.9426468
 [6,] 1.151819  0.00000000 0.21191299 0.9426468
 [7,] 1.151819  0.00000000 0.21191299 0.9426468
```

```
 [8,] 1.151819  0.00000000 0.21191299 0.9426468
 [9,] 1.151819  0.00000000 0.21191299 0.9426468
[10,] 1.151819  0.00000000 0.21191299 0.9426468
[11,] 1.151819  0.00000000 0.21191299 0.9426468
[12,] 1.151819  0.00000000 0.21191299 0.9426468
[13,] 1.151819  0.00000000 0.21191299 0.9426468
[14,] 1.157935  0.00000000 0.21832485 0.9605737
[15,] 1.157935  0.00000000 0.21832485 0.9605737
[16,] 1.157935  0.00000000 0.21832485 0.9605737
[17,] 1.155223  0.00000000 0.21921019 0.9550434
[18,] 1.155223  0.00000000 0.21921019 0.9550434
[19,] 1.174764 -0.03847477 0.23268823 0.9606195
[20,] 1.174764 -0.03847477 0.23268823 0.9606195
[21,] 1.174764 -0.03847477 0.23268823 0.9606195
[22,] 1.174764 -0.03847477 0.23268823 0.9606195
[23,] 1.174764 -0.03847477 0.23268823 0.9606195
```

```
> lr_output$S1.PROB
```

```
 [1] 1.0 1.0 0.8 0.5 1.0 1.0 1.0 0.5 1.0 1.0
```

```
> lr_output$S2.PROB
```

```
 [1] 1 1 1 1 1 1 1 1 1 1
```

In the output, there is a list of output values including Bootstrap imputated dataset(Bimp), Final varaible selected indicator(S.fin), final parameter estimates for the regression(beta.fin), final selection probability for lasso (S1.prob), final selection for variable selection indicator for stability selection with randomized lasso(S2.prob)
Output row format:(row number) 1 Direct use of linear/ generalized linear regression
2 naive estimate using only complete observations and lasso
3 naive estimate using only complete observations and adaptive lasso
4-8 Direct use of lasso <threshold: pct=c(0.6,0.7,0.8,0.9,1.0);> (the final estimate of beta using mean of beta's from the previous step, stability selection with randomized lasso)
9-13 Direct use of stability selection with randomized lasso (the final estimate of beta using mean of beta's from the previous step, stability selection with randomized lasso)
14-18 Indirect use of lasso: First conduct feature selection using bootstrap lasso, then rerun the regression analysis using only selected X's <threshold: pct=c(0.6,0.7,0.8,0.9,1.0);>
19-23 Indirect use of stability selection with randomized lasso: alternative calculation of beta by rerun the regression analysis using only selected X's <threshold: pct=c(0.6,0.7,0.8,0.9,1.0);>

2.2 Binary data

Previous examples are for Gaussian data and next we will focus on the case that it is binary variable that has missing values.

```
> dim(binaryData)

[1] 250  11

> binaryData[1:20,1:6]

   Y          X1          X2          X3          X4          X5
1  1          NA -0.38239954  2.23987455  0.58207947  1.03022710
2  0          NA -0.68623953 -1.42028968 -1.31297962 -0.79433605
3  1          NA  0.87064815  0.40136174 -0.21534102  0.84962787
4  0          NA  0.96470203  1.30535720 -0.54739009  0.93955738
5  1          NA -0.45778682  0.72868730  1.14481075  2.04716810
6  1 -0.39688067  1.23534424  0.52483873  0.36827968 -0.77277322
7  1          NA  0.04475838 -0.30954230  0.44060482  0.54967377
8  0 -0.88750540  0.98085239  0.80662866  0.64399419 -0.41387184
9  1 -0.07683133 -1.82916412 -1.24108147  0.15798779  0.45819317
10 0          NA -3.01530412 -2.31938361  1.29179688  1.45635020
11 1          NA  0.18420068  1.01051574  0.28663553 -0.23822022
12 1          NA  0.27571273  1.15433804  2.32489829  1.80216773
13 0  0.02925003 -0.05280988  0.18911768  0.03260646  0.31169122
14 1          NA  0.75189219 -1.72502506 -0.25553791 -0.16572650
15 1          NA -0.24029326  0.05384884 -0.71032932  1.20298066
16 1          NA  0.66078562  0.45584071 -0.12771327 -0.23736481
17 0 -1.84318579 -1.78648951 -0.96116580  0.99945948 -0.39015465
18 1 -0.04642971  0.03410440 -0.43048817 -0.05497932 -0.64927149
19 0  0.53899087  0.16824073  1.53860155  0.42827949  0.65846049
20 1 -0.63056726  0.57803965  0.46544429 -0.24895439 -0.06211272

> sum(is.na(binaryData))

[1] 118

> sum(is.na(binaryData))/dim(binaryData)[1]

[1] 0.472
```

Similar to *gaussianData*, only first column of this dataset has missing values.
The output list and format is the same as the guassian case.

```
> lg_output=BI_SS(binaryData[,-1],binaryData$Y,family="binomial",link=NULL,
+                 missing_col=1,MI.method="mice",BI.size=10,pi=NULL,nsteps=12)
> lg_output$BIMP[1,1:20,1:6]

             [,1]        [,2]        [,3]        [,4]        [,5]        [,6]
 [1,]  0.54523574 -0.38239954  2.23987455  0.58207947  1.0302271  1.31444187
 [2,]  0.28563553  0.87064815  0.40136174 -0.21534102  0.8496279  1.32630069
 [3,] -0.06751796  0.96470203  1.30535720 -0.54739009  0.9395574  0.95769987
 [4,] -0.39688067  1.23534424  0.52483873  0.36827968 -0.7727732  0.36447846
 [5,] -0.39688067  1.23534424  0.52483873  0.36827968 -0.7727732  0.36447846
 [6,]  2.44181760  0.04475838 -0.30954230  0.44060482  0.5496738 -0.43914172
 [7,] -0.88750540  0.98085239  0.80662866  0.64399419 -0.4138718  0.83823338
 [8,] -0.34663589 -3.01530412 -2.31938361  1.29179688  1.4563502  0.04490134
```

```
 [9,]  0.24674889 -3.01530412 -2.31938361  1.29179688  1.4563502  0.04490134
[10,]  0.97503173  0.18420068  1.01051574  0.28663553 -0.2382202  0.71971055
[11,]  1.55872083  0.27571273  1.15433804  2.32489829  1.8021677  1.02098753
[12,]  0.02925003 -0.05280988  0.18911768  0.03260646  0.3116912  0.65701738
[13,]  0.02925003 -0.05280988  0.18911768  0.03260646  0.3116912  0.65701738
[14,]  0.02925003 -0.05280988  0.18911768  0.03260646  0.3116912  0.65701738
[15,]  1.02107545  0.75189219 -1.72502506 -0.25553791 -0.1657265  0.72719538
[16,]  2.44181760  0.75189219 -1.72502506 -0.25553791 -0.1657265  0.72719538
[17,]  2.44181760  0.75189219 -1.72502506 -0.25553791 -0.1657265  0.72719538
[18,] -0.39688067 -0.24029326  0.05384884 -0.71032932  1.2029807  0.89801012
[19,] -0.39688067  0.66078562  0.45584071 -0.12771327 -0.2373648  0.33088282
[20,] -0.04642971  0.03410440 -0.43048817 -0.05497932 -0.6492715  0.43123106
```

> lg_output$S.FIN

```
      [,1] [,2] [,3] [,4] [,5] [,6] [,7] [,8] [,9] [,10]
 [1,]    1    1    1    1    1    1    1    1    1     1
 [2,]    1    1    0    1    1    1    0    0    0     1
 [3,]    1    1    0    0    1    1    0    0    0     0
 [4,]    1    1    1    1    1    1    1    0    1     1
 [5,]    1    1    1    0    1    1    1    0    1     1
 [6,]    1    1    0    0    1    1    1    0    1     1
 [7,]    1    1    0    0    1    1    1    0    1     1
 [8,]    1    1    0    0    1    1    1    0    1     0
 [9,]    1    1    1    1    1    1    1    1    1     1
[10,]    1    1    1    1    1    1    1    1    1     1
[11,]    1    1    1    1    1    1    1    1    1     1
[12,]    1    1    1    1    1    1    1    1    1     1
[13,]    1    1    1    1    1    1    1    1    1     1
[14,]    1    1    1    1    1    1    1    0    1     1
[15,]    1    1    1    0    1    1    1    0    1     1
[16,]    1    1    0    0    1    1    1    0    1     1
[17,]    1    1    0    0    1    1    1    0    1     1
[18,]    1    1    0    0    1    1    1    0    1     0
[19,]    1    1    1    1    1    1    1    1    1     1
[20,]    1    1    1    1    1    1    1    1    1     1
[21,]    1    1    1    1    1    1    1    1    1     1
[22,]    1    1    1    1    1    1    1    1    1     1
[23,]    1    1    1    1    1    1    1    1    1     1
```

> lg_output$BETA.FIN

```
            [,1]       [,2]      [,3]         [,4]        [,5]      [,6]
 [1,] -0.99791738 1.19392302 1.4699954 -0.419656615  0.58776402 0.8841016
 [2,] -0.86134021 0.77408463 0.9317345  0.000000000  0.26276553 0.6915423
 [3,]  0.37469198 0.07275653 0.1324558  0.000000000  0.00000000 0.1147306
 [4,]  0.02138222 0.81234189 1.0078643 -0.085541332 -0.05754759 0.7460548
 [5,]  0.02138222 0.81234189 1.0078643 -0.085541332  0.00000000 0.7460548
 [6,]  0.02138222 0.81234189 1.0078643  0.000000000  0.00000000 0.7460548
 [7,]  0.02138222 0.81234189 1.0078643  0.000000000  0.00000000 0.7460548
 [8,]  0.02138222 0.81234189 1.0078643  0.000000000  0.00000000 0.7460548
```

```
 [9,]  0.02138222 0.81234189 1.0078643 -0.085541332 -0.05754759 0.7460548
[10,]  0.02138222 0.81234189 1.0078643 -0.085541332  0.00000000 0.7460548
[11,]  0.02138222 0.81234189 1.0078643  0.000000000  0.00000000 0.7460548
[12,]  0.02138222 0.81234189 1.0078643  0.000000000  0.00000000 0.7460548
[13,]  0.02138222 0.81234189 1.0078643  0.000000000  0.00000000 0.7460548
[14,]  0.50771024 0.09569535 0.1449598 -0.009092387 -0.02318825 0.1176471
[15,]  0.50819255 0.09360578 0.1473608 -0.018219629  0.00000000 0.1062426
[16,]  0.50702867 0.09312622 0.1407961  0.000000000  0.00000000 0.1042450
[17,]  0.50702867 0.09312622 0.1407961  0.000000000  0.00000000 0.1042450
[18,]  0.50567432 0.09213127 0.1400023  0.000000000  0.00000000 0.1036914
[19,]  0.50570950 0.09676965 0.1447353 -0.012252794 -0.01757900 0.1164303
[20,]  0.50570950 0.09676965 0.1447353 -0.012252794 -0.01757900 0.1164303
[21,]  0.50570950 0.09676965 0.1447353 -0.012252794 -0.01757900 0.1164303
[22,]  0.50570950 0.09676965 0.1447353 -0.012252794 -0.01757900 0.1164303
[23,]  0.50570950 0.09676965 0.1447353 -0.012252794 -0.01757900 0.1164303
            [,7]       [,8]        [,9]      [,10]      [,11]
 [1,] 1.33516166 0.24351596 -0.52516950 0.11968298 0.49263113
 [2,] 0.87703280 0.00000000  0.00000000 0.00000000 0.19325553
 [3,] 0.11053812 0.00000000  0.00000000 0.00000000 0.00000000
 [4,] 0.65443790 0.60475562  0.00000000 0.54620808 0.25333434
 [5,] 0.65443790 0.60475562  0.00000000 0.54620808 0.25333434
 [6,] 0.65443790 0.60475562  0.00000000 0.54620808 0.25333434
 [7,] 0.65443790 0.60475562  0.00000000 0.54620808 0.25333434
 [8,] 0.65443790 0.60475562  0.00000000 0.54620808 0.00000000
 [9,] 0.65443790 0.60475562  0.00000000 0.54620808 0.25333434
[10,] 0.65443790 0.60475562  0.00000000 0.54620808 0.25333434
[11,] 0.65443790 0.60475562  0.00000000 0.54620808 0.25333434
[12,] 0.65443790 0.60475562  0.00000000 0.54620808 0.25333434
[13,] 0.65443790 0.60475562  0.00000000 0.54620808 0.00000000
[14,] 0.09284131 0.08574441  0.00000000 0.07022697 0.03713966
[15,] 0.09261586 0.08712282  0.00000000 0.07182560 0.03665787
[16,] 0.09086140 0.08669443  0.00000000 0.07038788 0.03813973
[17,] 0.09086140 0.08669443  0.00000000 0.07038788 0.03813973
[18,] 0.09114525 0.08665073  0.00000000 0.09014356 0.00000000
[19,] 0.08776331 0.09914338 -0.03730419 0.08359644 0.03858441
[20,] 0.08776331 0.09914338 -0.03730419 0.08359644 0.03858441
[21,] 0.08776331 0.09914338 -0.03730419 0.08359644 0.03858441
[22,] 0.08776331 0.09914338 -0.03730419 0.08359644 0.03858441
[23,] 0.08776331 0.09914338 -0.03730419 0.08359644 0.03858441
```

```
> lg_output$S1.PROB
```

```
 [1] 1.0 1.0 0.7 0.6 1.0 1.0 1.0 0.5 1.0 0.9
```

```
> lg_output$S2.PROB
```

```
 [1] 1 1 1 1 1 1 1 1 1 1
```

2.3 Poisson data

Previous examples are for Binary data and next we will focus on the case that it is poisson variable that has missing values.

```
> dim(poissonData)

[1] 250  11

> poissonData[1:20,1:6]

      Y          X1          X2          X3          X4          X5
1     4          NA -0.38239954  2.23987455  0.58207947  1.03022710
2     0  0.98179734 -0.68623953 -1.42028968 -1.31297962 -0.79433605
3   294          NA  0.87064815  0.40136174 -0.21534102  0.84962787
4    47          NA  0.96470203  1.30535720 -0.54739009  0.93955738
5  1012          NA -0.45778682  0.72868730  1.14481075  2.04716810
6     1 -0.39688067  1.23534424  0.52483873  0.36827968 -0.77277322
7     4          NA  0.04475838 -0.30954230  0.44060482  0.54967377
8     3          NA  0.98085239  0.80662866  0.64399419 -0.41387184
9     3          NA -1.82916412 -1.24108147  0.15798779  0.45819317
10    0 -0.79543141 -3.01530412 -2.31938361  1.29179688  1.45635020
11   23          NA  0.18420068  1.01051574  0.28663553 -0.23822022
12   25          NA  0.27571273  1.15433804  2.32489829  1.80216773
13    5          NA -0.05280988  0.18911768  0.03260646  0.31169122
14   12          NA  0.75189219 -1.72502506 -0.25553791 -0.16572650
15    5          NA -0.24029326  0.05384884 -0.71032932  1.20298066
16   46          NA  0.66078562  0.45584071 -0.12771327 -0.23736481
17    0 -1.84318579 -1.78648951 -0.96116580  0.99945948 -0.39015465
18    0 -0.04642971  0.03410440 -0.43048817 -0.05497932 -0.64927149
19    0  0.53899087  0.16824073  1.53860155  0.42827949  0.65846049
20    5          NA  0.57803965  0.46544429 -0.24895439 -0.06211272

> sum(is.na(poissonData))

[1] 151

> sum(is.na(poissonData))/dim(poissonData)[1]

[1] 0.604
```

Similar to *gaussianData*, only first column of this dataset has missing values.
The output list and format is the same as the guassian case.

```
> po_output=BI_SS(poissonData[,-1],poissonData$Y,family="poisson",link=NULL,
+                 missing_col=1,MI.method="mice",BI.size=10,pi=NULL,nsteps=12)
> po_output$BIMP[1,1:20,1:6]

            [,1]        [,2]        [,3]        [,4]       [,5]       [,6]
 [1,] -0.20629018 -0.38239954  2.23987455  0.58207947  1.0302271  1.3144419
 [2,] -0.20629018 -0.38239954  2.23987455  0.58207947  1.0302271  1.3144419
 [3,]  0.98179734 -0.68623953 -1.42028968 -1.31297962 -0.7943360 -1.4401720
 [4,]  0.29644711  0.87064815  0.40136174 -0.21534102  0.8496279  1.3263007
 [5,]  0.01941497  0.96470203  1.30535720 -0.54739009  0.9395574  0.9576999
 [6,] -0.39688067  1.23534424  0.52483873  0.36827968 -0.7727732  0.3644785
 [7,] -0.39688067  1.23534424  0.52483873  0.36827968 -0.7727732  0.3644785
 [8,]  0.01941497  0.04475838 -0.30954230  0.44060482  0.5496738 -0.4391417
```

```
 [9,] -0.20629018  0.04475838 -0.30954230  0.44060482  0.5496738 -0.4391417
[10,]  0.01941497  0.04475838 -0.30954230  0.44060482  0.5496738 -0.4391417
[11,]  0.01941497 -1.82916412 -1.24108147  0.15798779  0.4581932  1.0159846
[12,]  0.01941497 -1.82916412 -1.24108147  0.15798779  0.4581932  1.0159846
[13,]  0.29644711  0.18420068  1.01051574  0.28663553 -0.2382202  0.7197105
[14,]  0.01941497  0.27571273  1.15433804  2.32489829  1.8021677  1.0209875
[15,]  0.29644711  0.27571273  1.15433804  2.32489829  1.8021677  1.0209875
[16,]  0.29644711  0.27571273  1.15433804  2.32489829  1.8021677  1.0209875
[17,]  0.01941497 -0.05280988  0.18911768  0.03260646  0.3116912  0.6570174
[18,]  0.29644711  0.75189219 -1.72502506 -0.25553791 -0.1657265  0.7271954
[19,] -0.41712382 -0.24029326  0.05384884 -0.71032932  1.2029807  0.8980101
[20,] -1.84318579 -1.78648951 -0.96116580  0.99945948 -0.3901547  0.9359850
```

> po_output$S.FIN

```
      [,1] [,2] [,3] [,4] [,5] [,6] [,7] [,8] [,9] [,10]
 [1,]    1    1    1    1    1    1    1    1    1     1
 [2,]    1    1    1    1    1    1    1    1    1     1
 [3,]    1    1    1    1    1    1    1    1    1     1
 [4,]    0    1    1    1    1    1    1    1    1     1
 [5,]    0    1    1    1    1    1    1    1    1     1
 [6,]    0    1    0    1    1    1    1    0    1     1
 [7,]    0    1    0    0    1    1    1    0    0     1
 [8,]    0    1    0    0    1    1    1    0    0     1
 [9,]    1    1    1    1    1    1    1    1    1     1
[10,]    1    1    1    1    1    1    1    1    1     1
[11,]    1    1    1    1    1    1    1    1    1     1
[12,]    1    1    1    1    1    1    1    1    1     1
[13,]    1    1    1    1    1    1    1    1    1     1
[14,]    0    1    1    1    1    1    1    1    1     1
[15,]    0    1    1    1    1    1    1    1    1     1
[16,]    0    1    0    1    1    1    1    0    1     1
[17,]    0    1    0    0    1    1    1    0    0     1
[18,]    0    1    0    0    1    1    1    0    0     1
[19,]    1    1    1    1    1    1    1    1    1     1
[20,]    1    1    1    1    1    1    1    1    1     1
[21,]    1    1    1    1    1    1    1    1    1     1
[22,]    1    1    1    1    1    1    1    1    1     1
[23,]    1    1    1    1    1    1    1    1    1     1
```

> po_output$BETA.FIN

```
           [,1]        [,2]        [,3]         [,4]         [,5]        [,6]
 [1,] -0.6090430   1.3287394  0.92341372  -0.24420992 -0.002208643  0.88861057
 [2,] -0.5707277   1.1853340  0.66680620  -0.06047975  0.002853161  0.73918003
 [3,]  0.5447040   0.1835499  0.09595536  -0.03980876  0.043158172  0.08393642
 [4,]  0.2865508   0.0000000  1.29736006   0.10898571 -0.061145876  1.18053996
 [5,]  0.2865508   0.0000000  1.29736006   0.10898571 -0.061145876  1.18053996
 [6,]  0.2865508   0.0000000  1.29736006   0.00000000 -0.061145876  1.18053996
 [7,]  0.2865508   0.0000000  1.29736006   0.00000000  0.000000000  1.18053996
 [8,]  0.2865508   0.0000000  1.29736006   0.00000000  0.000000000  1.18053996
```

```
 [9,]  0.2865508   0.0000000  1.29736006   0.10898571 -0.061145876  1.18053996
[10,]  0.2865508   0.0000000  1.29736006   0.10898571 -0.061145876  1.18053996
[11,]  0.2865508   0.0000000  1.29736006   0.00000000 -0.061145876  1.18053996
[12,]  0.2865508   0.0000000  1.29736006   0.00000000  0.000000000  1.18053996
[13,]  0.2865508   0.0000000  1.29736006   0.00000000  0.000000000  1.18053996
[14,] 79.2898315   0.0000000 80.82961196 -28.16860356 17.562068507 36.49451113
[15,] 79.2898315   0.0000000 80.82961196 -28.16860356 17.562068507 36.49451113
[16,] 77.0702503   0.0000000 69.00834245   0.00000000  9.972842703 36.37699759
[17,] 77.5298220   0.0000000 70.26794432   0.00000000  0.000000000 41.69545707
[18,] 77.5298220   0.0000000 70.26794432   0.00000000  0.000000000 41.69545707
[19,] 83.2650537 -18.3538750 87.28129248 -29.34944454 19.806913000 38.49718210
[20,] 83.2650537 -18.3538750 87.28129248 -29.34944454 19.806913000 38.49718210
[21,] 83.2650537 -18.3538750 87.28129248 -29.34944454 19.806913000 38.49718210
[22,] 83.2650537 -18.3538750 87.28129248 -29.34944454 19.806913000 38.49718210
[23,] 83.2650537 -18.3538750 87.28129248 -29.34944454 19.806913000 38.49718210
            [,7]       [,8]        [,9]       [,10]      [,11]
 [1,]  0.7392063  1.3872704 -0.51792294  0.25653965  0.9296146
 [2,]  0.6002150  1.1506992 -0.29640148  0.10057732  0.7768629
 [3,]  0.0992403  0.1846163 -0.06801437  0.04520117  0.1200194
 [4,]  0.9030950  1.1120873  0.09028904  0.01601677  0.9639825
 [5,]  0.9030950  1.1120873  0.09028904  0.01601677  0.9639825
 [6,]  0.9030950  1.1120873  0.00000000  0.01601677  0.9639825
 [7,]  0.9030950  1.1120873  0.00000000  0.00000000  0.9639825
 [8,]  0.9030950  1.1120873  0.00000000  0.00000000  0.9639825
 [9,]  0.9030950  1.1120873  0.09028904  0.01601677  0.9639825
[10,]  0.9030950  1.1120873  0.09028904  0.01601677  0.9639825
[11,]  0.9030950  1.1120873  0.00000000  0.01601677  0.9639825
[12,]  0.9030950  1.1120873  0.00000000  0.00000000  0.9639825
[13,]  0.9030950  1.1120873  0.00000000  0.00000000  0.9639825
[14,] 73.4214522 32.0446558  9.06319191 44.68749893 43.3615758
[15,] 73.4214522 32.0446558  9.06319191 44.68749893 43.3615758
[16,] 70.8280665 33.8528381  0.00000000 48.00552709 42.3763267
[17,] 68.3624124 42.3077882  0.00000000  0.00000000 64.6192588
[18,] 68.3624124 42.3077882  0.00000000  0.00000000 64.6192588
[19,] 76.6142301 34.4421738  8.99147344 46.20663981 42.7702282
[20,] 76.6142301 34.4421738  8.99147344 46.20663981 42.7702282
[21,] 76.6142301 34.4421738  8.99147344 46.20663981 42.7702282
[22,] 76.6142301 34.4421738  8.99147344 46.20663981 42.7702282
[23,] 76.6142301 34.4421738  8.99147344 46.20663981 42.7702282

> po_output$S1.PROB

 [1] 0.5 1.0 0.7 0.8 1.0 1.0 1.0 0.7 0.8 1.0

> po_output$S2.PROB

 [1] 1 1 1 1 1 1 1 1 1 1

>
```

2.4 Gaussian data with generalized missing pattern

Next we will focus on the case that the data have generalized missing pattern.

```
> dim(gaussianGeneralizedData)

[1] 200  41

> gaussianGeneralizedData[1:20,1:6]

             Y         X1           X2           X3          X4          X5
1   2.06550958 -0.6712193           NA  0.259543319  0.57995009 -0.25254570
2   0.95328035  1.0694551  0.373237489 -0.251923310  0.44102166 -0.94982572
3   0.82302915  1.2275720 -0.652890073           NA -1.49868902 -0.66789919
4   5.00267594         NA           NA           NA -0.04149170  1.30873245
5   6.32175315         NA           NA           NA -0.29569573 -0.14412405
6   3.52021750         NA  0.475416241           NA -0.09536167 -0.11499557
7  -3.59926179 -1.5172420 -1.205822656  0.784534956  0.05508580 -0.50487957
8   0.44162282 -0.4290579 -0.215869766 -1.056794848 -0.37486866 -1.17844843
9   1.02968899  1.1445852  1.081692044  0.883632096  1.28985684 -0.84842250
10 -2.31290659  0.5865488  0.006214288 -0.001641823 -0.39376429 -0.87679616
11  4.22142157         NA           NA           NA -1.22879152  1.44778160
12 -0.05379246 -0.4439172 -0.774007484 -1.190607207 -0.26286743 -0.55682087
13  3.23323884         NA           NA           NA -0.71991186  0.01710942
14  3.99735584         NA           NA           NA -0.85916671  0.78738112
15  3.55505863         NA  1.510992009           NA  0.10236392  0.40169736
16  4.29779621         NA           NA           NA -0.51070982  1.68203815
17  0.37125172 -1.0927519           NA  0.132552729  0.55920769  1.18400838
18  1.34130319         NA  1.260703140  0.547969162 -1.15737253 -0.84957226
19  1.91554850 -1.1826192  0.546243788  0.227770948  0.71149855 -0.69063994
20  3.97312267         NA -0.215648496  0.104724274  0.08143385  0.43179415

> missing=dim(gaussianGeneralizedData)[1]-sum(complete.cases(gaussianGeneralizedData))
> missing

[1] 104

> missing/dim(gaussianGeneralizedData)[1]

[1] 0.52
```

First three columns of this dataset have missing values. The output list and format is the same as the guassian case.

```
> gg_output=BI_SS(gaussianGeneralizedData[,-1],gaussianGeneralizedData$Y,family="gaussian"
+                 link=NULL,missing_col=1,MI.method="mice",BI.size=10,pi=NULL,nsteps=12)
> gg_output$BIMP[1,1:20,1:6]

           [,1]         [,2]         [,3]        [,4]        [,5]        [,6]
 [1,] -0.6712193  0.610281292  0.259543319  0.57995009 -0.25254570 -0.57231777
 [2,]  1.0694551  0.373237489 -0.251923310  0.44102166 -0.94982572 -0.18638426
 [3,]  1.2275720 -0.652890073 -1.648861840 -1.49868902 -0.66789919  0.91117069
 [4,]  1.6071101  0.610281292 -0.330207677 -0.29569573 -0.14412405  0.66004944
```

```
 [5,]  1.3030237  0.610281292 -1.082741910 -0.29569573 -0.14412405  0.66004944
 [6,] -1.5172420 -1.205822656  0.784534956  0.05508580 -0.50487957 -0.06167087
 [7,] -0.4290579 -0.215869766 -1.056794848 -0.37486866 -1.17844843  0.42043110
 [8,]  1.1445852  1.081692044  0.883632096  1.28985684 -0.84842250 -1.03023025
 [9,]  0.5865488  0.006214288 -0.001641823 -0.39376429 -0.87679616  0.26252352
[10,] -0.4439172 -0.774007484 -1.190607207 -0.26286743 -0.55682087  0.12927480
[11,] -0.4439172 -0.774007484 -1.190607207 -0.26286743 -0.55682087  0.12927480
[12,] -1.4909842  0.005704810 -1.053309669 -0.71991186  0.01710942  0.38847833
[13,]  1.6071101  0.610281292  0.253383282 -0.85916671  0.78738112 -0.18442309
[14,]  1.2028447  1.393433762  1.105478660 -0.85916671  0.78738112 -0.18442309
[15,]  1.3030237  1.510992009  0.158221119  0.10236392  0.40169736  0.27120534
[16,]  0.3157399  1.260703140  0.547969162 -1.15737253 -0.84957226 -0.34702573
[17,] -1.1826192  0.546243788  0.227770948  0.71149855 -0.69063994  0.33957406
[18,] -1.1826192  0.546243788  0.227770948  0.71149855 -0.69063994  0.33957406
[19,] -1.1826192  0.546243788  0.227770948  0.71149855 -0.69063994  0.33957406
[20,]  1.2028447 -0.215648496  0.104724274  0.08143385  0.43179415  1.37188213
```

> *gg_output$S.FIN*

```
      [,1] [,2] [,3] [,4] [,5] [,6] [,7] [,8] [,9] [,10] [,11] [,12] [,13]
 [1,]    1    1    1    1    1    1    1    1    1     1     1     1     1
 [2,]    1    1    0    1    1    1    1    0    0     1     0     1     0
 [3,]    1    1    0    0    1    1    1    0    0     1     0     0     0
 [4,]    1    1    1    1    1    1    1    1    1     1     1     1     1
 [5,]    1    1    1    1    1    1    1    1    1     1     1     1     1
 [6,]    1    1    1    1    1    1    1    1    1     1     0     1     0
 [7,]    1    1    1    1    1    1    1    0    1     1     0     0     0
 [8,]    1    1    1    1    1    1    1    0    1     1     0     0     0
 [9,]    1    1    1    1    1    1    1    1    1     1     1     1     1
[10,]    1    1    1    1    1    1    1    1    1     1     1     1     1
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[14,]    1    1    1    1    1    1    1    1    1     1     1     1     1
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[18,]    1    1    1    1    1    1    1    0    1     1     0     0     0
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[20,]    1    1    1    1    1    1    1    1    1     1     1     1     1
[21,]    1    1    1    1    1    1    1    1    1     1     1     1     1
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[23,]    1    1    1    1    1    1    1    1    1     1     1     1     1
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[11,]     1     1     1     1     1     1     1     1     1     1     1     1
[12,]     1     1     1     1     1     1     1     1     1     1     1     1
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[19,]     1     1     1     1     1     1     1     1     1     1     1     1
[20,]     1     1     1     1     1     1     1     1     1     1     1     1
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[23,]     1     1     1     1     1     1     1     1     1     1     1     1
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 [3,]     0     0     1     0     0     0     0     0     0     0     0     0
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[23,]     1     1     1     1     1     1     1     1     1     1     1     1
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 [2,]     1     0     1
 [3,]     1     0     0
 [4,]     1     1     1
 [5,]     1     1     1
 [6,]     1     1     1
 [7,]     1     1     0
 [8,]     1     0     0
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 [9,]     1     1     1
[10,]     1     1     1
[11,]     1     1     1
[12,]     1     1     1
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[15,]     1     1     1
[16,]     1     1     1
[17,]     1     1     0
[18,]     1     0     0
[19,]     1     1     1
[20,]     1     1     1
[21,]     1     1     1
[22,]     1     1     1
[23,]     1     1     1
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> gg_output$BETA.FIN

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 [3,] 0.3167278 0.6802748 0.8986412 0.00000000  0.00000000 1.011674 0.8491752
 [4,] 1.2133976 1.1321793 1.0760619 0.22064230 -0.27868432 1.349098 1.2040409
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[13,] 1.2133976 1.1321793 1.0760619 0.22064230 -0.27868432 1.349098 1.2040409
[14,] 1.2618865 1.2555263 1.0656081 0.25151640 -0.34237559 1.383607 1.2454662
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[16,] 1.2378682 1.2045691 1.0687657 0.22004770 -0.33284999 1.353739 1.2283777
[17,] 1.2043022 1.0803249 1.1455351 0.22778068 -0.36028738 1.391173 1.1729564
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 [3,] -0.29919488  0.00000000 0.0000000 0.8974824  0.000000000  0.00000000
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[16,] 0.25882902  0.08726685 -0.2317590 0.43347313 -0.1950011 0.06331931
[17,] 0.26531147  0.07107525 -0.2161514 0.41747033 -0.2041587 0.06831545
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 [3,] 0.000000000 0.00000000  0.00000000 0.00000000  0.0000000  0.000000000
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```

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[16,] 0.070392886 0.06200877 -0.11785855 0.23932816 -0.2279106 -0.003765705
[17,] 0.025814571 0.09978852 -0.10840530 0.24745583 -0.2301101  0.007175642
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 [5,] 0.13243642 -0.36441247  0.21211463 0.06522396
 [6,] 0.13243642 -0.36441247  0.21211463 0.06522396
 [7,] 0.13243642 -0.36441247  0.21211463 0.00000000
 [8,] 0.13243642 -0.36441247  0.00000000 0.00000000
 [9,] 0.13243642 -0.36441247  0.21211463 0.06522396
[10,] 0.13243642 -0.36441247  0.21211463 0.06522396
[11,] 0.13243642 -0.36441247  0.21211463 0.06522396
[12,] 0.13243642 -0.36441247  0.21211463 0.00000000
[13,] 0.13243642 -0.36441247  0.00000000 0.00000000
[14,] 0.17266055 -0.40771890  0.23764728 0.08717672
[15,] 0.17266055 -0.40771890  0.23764728 0.08717672
[16,] 0.14396372 -0.37781685  0.24249408 0.08388855
[17,] 0.12589368 -0.38423600  0.28753504 0.00000000
[18,] 0.07350468 -0.22643655  0.00000000 0.00000000
[19,] 0.17266055 -0.40771890  0.23764728 0.08717672
[20,] 0.17266055 -0.40771890  0.23764728 0.08717672
[21,] 0.17266055 -0.40771890  0.23764728 0.08717672
[22,] 0.17266055 -0.40771890  0.23764728 0.08717672
[23,] 0.17266055 -0.40771890  0.23764728 0.08717672
```

```
> gg_output$S1.PROB
```

```
 [1] 1.0 1.0 1.0 1.0 1.0 1.0 1.0 0.8 1.0 1.0 0.7 0.8 0.7 1.0 1.0 0.9 1.0 0.9 0.9
[20] 0.9 1.0 0.8 0.7 1.0 1.0 1.0 0.9 1.0 1.0 0.9 0.9 1.0 1.0 1.0 1.0 1.0 1.0 1.0
[39] 0.9 0.8
```

```
> gg_output$S2.PROB
```

```
 [1] 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
[39] 1 1
```

2.5 High dimensional gaussian data

Previous examples are for low dimensional data and next we will focus on the case that it is high dimensional data that have missing values.

```
> dim(gaussianHighDimensionData)

[1] 200 401

> gaussianHighDimensionData[1:20,1:6]

             Y         X1         X2          X3          X4          X5
1    9.4884955         NA -0.2873574  1.40073846  1.10218419  1.10303304
2  -12.1781888 -0.5400843 -0.6940944 -1.45186353 -1.08897227 -0.41957648
3   -1.2973931  0.7897745  1.2400397 -0.13073349 -0.69513719 -0.04545116
4    4.1611507         NA  1.6252387 -0.45383922  1.00084783  0.90519876
5   -0.7545917  0.4228495 -0.3617921  0.85790475  0.09930185 -1.02999112
6   -7.0865207  0.2066970 -0.2986331  0.86124248 -0.44218702  0.51093535
7    3.2136976         NA  0.6559665  2.58309403  2.44758083  1.21165686
8    0.8594289         NA -0.5403639 -0.15563857  1.61995496 -0.01936032
9   -7.2191238  2.1471170 -0.2423663  0.57118826  1.50821181  0.83984491
10   0.8037767 -0.3728233 -0.7294769  0.09669404 -0.21268971  1.65468558
11   7.1411261         NA -1.5637650 -0.81900236 -1.15036189 -0.63393790
12   5.2041949         NA  0.7533645 -0.29724578 -0.39585318 -1.16873774
13  -0.9026680 -0.8988914 -1.8288790 -0.79862101 -0.65550436 -0.38847272
14  -8.4392373 -1.7882416 -1.4437016 -2.13380051 -2.71006638 -1.35011041
15  -6.0725517 -0.1044651 -1.7139452 -1.06490711 -1.24580300 -1.15717572
16   3.7748304         NA -1.3179039 -1.38157792 -1.01272977  0.33632114
17   4.9985643         NA -1.7352829 -2.36611783 -1.53841756 -1.51377834
18  -6.1272997 -0.4919092 -1.7953581 -2.28709764 -2.23307494 -1.93453881
19  -0.6438643  0.7706034 -0.3426619 -0.81349500 -0.53483544  0.27952507
20   0.1841769         NA  0.3968575  0.44549843  1.26566889  0.84695093

> sum(is.na(gaussianHighDimensionData))

[1] 95

> sum(is.na(gaussianHighDimensionData))/dim(gaussianHighDimensionData)[1]

[1] 0.475
```

Similar to *gaussianData*, only first column of this dataset has missing values.
The output list and format is the same as the guassian case.

```
> h_output=BI_SS(gaussianHighDimensionData[,-1],gaussianHighDimensionData$Y,family="gaussi
+                link=NULL,missing_col=1,MI.method="blasso",BI.size=10,pi=NULL,nsteps=12)

Sampler Progress...
| 0%      20%      40%      60%      80%      100% |
|************************************************|


Sampler Progress...
| 0%      20%      40%      60%      80%      100% |
|************************************************|
```

```
Sampler Progress...
| 0%      20%      40%      60%      80%      100% |
|**************************************************|


Sampler Progress...
| 0%      20%      40%      60%      80%      100% |
|**************************************************|


Sampler Progress...
| 0%      20%      40%      60%      80%      100% |
|**************************************************|


Sampler Progress...
| 0%      20%      40%      60%      80%      100% |
|**************************************************|


Sampler Progress...
| 0%      20%      40%      60%      80%      100% |
|**************************************************|


Sampler Progress...
| 0%      20%      40%      60%      80%      100% |
|**************************************************|


Sampler Progress...
| 0%      20%      40%      60%      80%      100% |
|**************************************************|


Sampler Progress...
| 0%      20%      40%      60%      80%      100% |
|**************************************************|
```

> h_output$BIMP[1,1:20,1:6]

```
           [,1]       [,2]       [,3]        [,4]        [,5]        [,6]
 [1,] -0.5400843 -0.6940944 -1.4518635 -1.08897227 -0.41957648  0.59890590
 [2,]  0.7897745  1.2400397 -0.1307335 -0.69513719 -0.04545116  0.73033407
 [3,]  0.7897745  1.2400397 -0.1307335 -0.69513719 -0.04545116  0.73033407
 [4,]  0.7262209  1.6252387 -0.4538392  1.00084783  0.90519876  0.09216117
 [5,]  0.4228495 -0.3617921  0.8579048  0.09930185 -1.02999112 -0.36156660
 [6,]  0.4228495 -0.3617921  0.8579048  0.09930185 -1.02999112 -0.36156660
 [7,]  0.2066970 -0.2986331  0.8612425 -0.44218702  0.51093535  0.03483325
 [8,]  0.2066970 -0.2986331  0.8612425 -0.44218702  0.51093535  0.03483325
```

```
 [9,]  0.2066970 -0.2986331  0.8612425 -0.44218702  0.51093535  0.03483325
[10,]  5.9806368  0.6559665  2.5830940  2.44758083  1.21165686 -0.63486321
[11,]  6.5621167  0.6559665  2.5830940  2.44758083  1.21165686 -0.63486321
[12,]  5.7334405  0.6559665  2.5830940  2.44758083  1.21165686 -0.63486321
[13,]  1.1047925 -0.5403639 -0.1556386  1.61995496 -0.01936032  1.23998759
[14,]  0.6185755 -0.5403639 -0.1556386  1.61995496 -0.01936032  1.23998759
[15,]  0.3619641 -0.5403639 -0.1556386  1.61995496 -0.01936032  1.23998759
[16,]  0.8898829 -0.5403639 -0.1556386  1.61995496 -0.01936032  1.23998759
[17,]  2.1471170 -0.2423663  0.5711883  1.50821181  0.83984491  0.96551951
[18,] -1.1212210  0.7533645 -0.2972458 -0.39585318 -1.16873774 -1.54388772
[19,] -0.8988914 -1.8288790 -0.7986210 -0.65550436 -0.38847272  0.40759545
[20,] -1.7882416 -1.4437016 -2.1338005 -2.71006638 -1.35011041  0.81331656
```

```
> h_output$S.FIN[1:23,1:40]
```

```
      [,1] [,2] [,3] [,4] [,5] [,6] [,7] [,8] [,9] [,10] [,11] [,12] [,13]
 [1,]    0    0    0    0    0    0    0    0    0     0     0     0     0
 [2,]    1    1    0    1    1    1    0    0    0     1     1     1     1
 [3,]    1    1    0    1    1    1    0    0    0     1     1     1     0
 [4,]    1    1    0    0    1    1    0    0    0     1     1     1     0
 [5,]    1    1    0    0    1    1    0    0    0     1     1     1     0
 [6,]    1    1    0    0    1    1    0    0    0     1     1     1     0
 [7,]    0    1    0    0    1    1    0    0    0     1     1     1     0
 [8,]    0    1    0    0    1    1    0    0    0     1     1     1     0
 [9,]    1    1    1    1    1    0    0    0    0     0     0     1     0
[10,]    1    1    1    1    1    0    0    0    0     0     0     1     0
[11,]    1    1    1    1    0    0    0    0    0     0     0     0     0
[12,]    1    1    1    1    0    0    0    0    0     0     0     0     0
[13,]    1    1    1    1    0    0    0    0    0     0     0     0     0
[14,]    1    1    0    0    1    1    0    0    0     1     1     1     0
[15,]    1    1    0    0    1    1    0    0    0     1     1     1     0
[16,]    1    1    0    0    1    1    0    0    0     1     1     1     0
[17,]    0    1    0    0    1    1    0    0    0     1     1     1     0
[18,]    0    1    0    0    1    1    0    0    0     1     1     1     0
[19,]    1    1    1    1    1    0    0    0    0     0     0     1     0
[20,]    1    1    1    1    1    0    0    0    0     0     0     1     0
[21,]    1    1    1    1    0    0    0    0    0     0     0     0     0
[22,]    1    1    1    1    0    0    0    0    0     0     0     0     0
[23,]    1    1    1    1    0    0    0    0    0     0     0     0     0
      [,14] [,15] [,16] [,17] [,18] [,19] [,20] [,21] [,22] [,23] [,24] [,25]
 [1,]     0     0     0     0     0     0     0     0     0     0     0     0
 [2,]     0     1     1     1     0     1     1     1     1     0     1     1
 [3,]     0     1     1     0     0     0     1     1     1     0     0     1
 [4,]     0     1     1     1     0     0     1     1     1     0     0     1
 [5,]     0     1     1     1     0     0     1     1     1     0     0     1
 [6,]     0     1     1     0     0     0     1     1     1     0     0     1
 [7,]     0     1     1     0     0     0     1     1     1     0     0     1
 [8,]     0     1     1     0     0     0     1     1     1     0     0     1
 [9,]     0     0     1     0     0     1     1     1     0     0     0     1
[10,]     0     0     0     0     0     0     1     0     0     0     0     0
```

```
[11,]    0    0    0    0    0    0    1    0    0    0    0    0
[12,]    0    0    0    0    0    0    1    0    0    0    0    0
[13,]    0    0    0    0    0    0    0    0    0    0    0    0
[14,]    0    1    1    1    0    0    1    1    1    0    0    1
[15,]    0    1    1    1    0    0    1    1    1    0    0    1
[16,]    0    1    1    0    0    0    1    1    1    0    0    1
[17,]    0    1    1    0    0    0    1    1    1    0    0    1
[18,]    0    1    1    0    0    0    1    1    1    0    0    1
[19,]    0    0    1    0    0    1    1    1    0    0    0    1
[20,]    0    0    0    0    0    0    1    0    0    0    0    0
[21,]    0    0    0    0    0    0    1    0    0    0    0    0
[22,]    0    0    0    0    0    0    1    0    0    0    0    0
[23,]    0    0    0    0    0    0    0    0    0    0    0    0
      [,26] [,27] [,28] [,29] [,30] [,31] [,32] [,33] [,34] [,35] [,36] [,37]
 [1,]    0    0    0    0    0    0    0    0    0    0    0    0
 [2,]    1    0    0    0    1    1    1    1    0    1    1    0
 [3,]    1    0    0    0    1    1    1    1    0    1    1    0
 [4,]    1    0    0    0    1    1    1    0    0    1    1    1
 [5,]    1    0    0    0    1    1    1    0    0    1    1    1
 [6,]    1    0    0    0    1    1    1    0    0    1    1    1
 [7,]    1    0    0    0    1    1    1    0    0    1    1    1
 [8,]    1    0    0    0    1    1    1    0    0    1    1    0
 [9,]    0    1    1    0    0    1    1    0    1    1    1    1
[10,]    0    0    0    0    0    1    0    0    1    1    1    1
[11,]    0    0    0    0    0    1    0    0    1    1    1    1
[12,]    0    0    0    0    0    0    0    0    1    1    1    1
[13,]    0    0    0    0    0    0    0    0    1    1    1    1
[14,]    1    0    0    0    1    1    1    0    0    1    1    1
[15,]    1    0    0    0    1    1    1    0    0    1    1    1
[16,]    1    0    0    0    1    1    1    0    0    1    1    1
[17,]    1    0    0    0    1    1    1    0    0    1    1    1
[18,]    1    0    0    0    1    1    1    0    0    1    1    0
[19,]    0    1    1    0    0    1    1    0    1    1    1    1
[20,]    0    0    0    0    0    1    0    0    1    1    1    1
[21,]    0    0    0    0    0    1    0    0    1    1    1    1
[22,]    0    0    0    0    0    0    0    0    1    1    1    1
[23,]    0    0    0    0    0    0    0    0    1    1    1    1
      [,38] [,39] [,40]
 [1,]    0    0    0
 [2,]    0    0    1
 [3,]    0    0    1
 [4,]    0    0    1
 [5,]    0    0    1
 [6,]    0    0    1
 [7,]    0    0    1
 [8,]    0    0    1
 [9,]    1    1    0
[10,]    1    0    0
[11,]    0    0    0
[12,]    0    0    0
```

```
[13,]     0     0     0
[14,]     0     0     1
[15,]     0     0     1
[16,]     0     0     1
[17,]     0     0     1
[18,]     0     0     1
[19,]     1     1     0
[20,]     1     0     0
[21,]     0     0     0
[22,]     0     0     0
[23,]     0     0     0
```

> h_output$BETA.FIN[1:23,1:40]

```
             [,1]          [,2]      [,3]       [,4]        [,5]       [,6]
 [1,]          NA            NA        NA         NA          NA         NA
 [2,] -1.64039013 5.278098e-01 0.7508734  0.0000000   0.2957546 0.3371937
 [3,] -0.85915509 6.203266e-01 0.8455214  0.0000000   0.2527983 0.6086818
 [4,] -0.05873599 5.361817e-02 1.2135954  0.0000000   0.0000000 0.8038603
 [5,] -0.05873599 5.361817e-02 1.2135954  0.0000000   0.0000000 0.8038603
 [6,] -0.05873599 5.361817e-02 1.2135954  0.0000000   0.0000000 0.8038603
 [7,] -0.05873599 0.000000e+00 1.2135954  0.0000000   0.0000000 0.8038603
 [8,] -0.05873599 0.000000e+00 1.2135954  0.0000000   0.0000000 0.8038603
 [9,] -0.05873599 5.361817e-02 1.2135954  0.0000000   0.0000000 0.8038603
[10,] -0.05873599 5.361817e-02 1.2135954  0.0000000   0.0000000 0.8038603
[11,] -0.05873599 5.361817e-02 1.2135954  0.0000000   0.0000000 0.8038603
[12,] -0.05873599 0.000000e+00 1.2135954  0.0000000   0.0000000 0.8038603
[13,] -0.05873599 0.000000e+00 1.2135954  0.0000000   0.0000000 0.8038603
[14,] -0.07928433 2.674163e-02 1.3395598  0.0000000   0.0000000 0.8132346
[15,] -0.01384968 4.562714e-02 1.3796375  0.0000000   0.0000000 0.8566099
[16,]  0.04340226 5.236510e-02 1.4765203  0.0000000   0.0000000 0.9312674
[17,]  0.07333939 0.000000e+00 1.4968389  0.0000000   0.0000000 0.9603352
[18,]  0.05370140 0.000000e+00 1.4969658  0.0000000   0.0000000 0.9482793
[19,] -8.63110604 6.619121e-15 2.6925152  1.5234438 -11.5235807 9.1125038
[20,] -0.34507311 5.713672e-02 1.3934611 -0.5558010   0.9421231 0.6488301
[21,] -0.22010651 1.907328e-01 1.3714580 -1.3593202   1.6935856 0.0000000
[22,] -0.16994858 1.919589e-01 1.1192162 -0.8459205   1.7278193 0.0000000
[23,] -0.05893792 1.945519e-01 1.0299082 -0.9831930   1.6042130 0.0000000
          [,7] [,8] [,9] [,10]     [,11]     [,12]      [,13]      [,14] [,15]
 [1,]       NA   NA   NA    NA        NA        NA         NA         NA    NA
 [2,] 0.8214168    0    0     0 0.7571491 0.4856891  0.9541457 0.03180277     0
 [3,] 0.8828733    0    0     0 0.8982718 0.7443727  1.0238135 0.00000000     0
 [4,] 0.9523305    0    0     0 1.1039367 0.7171843  0.8967833 0.00000000     0
 [5,] 0.9523305    0    0     0 1.1039367 0.7171843  0.8967833 0.00000000     0
 [6,] 0.9523305    0    0     0 1.1039367 0.7171843  0.8967833 0.00000000     0
 [7,] 0.9523305    0    0     0 1.1039367 0.7171843  0.8967833 0.00000000     0
 [8,] 0.9523305    0    0     0 1.1039367 0.7171843  0.8967833 0.00000000     0
 [9,] 0.9523305    0    0     0 1.1039367 0.7171843  0.8967833 0.00000000     0
[10,] 0.9523305    0    0     0 1.1039367 0.7171843  0.8967833 0.00000000     0
[11,] 0.9523305    0    0     0 1.1039367 0.7171843  0.8967833 0.00000000     0
```

```
[12,] 0.9523305    0    0     0 1.1039367 0.7171843  0.8967833 0.00000000     0
[13,] 0.9523305    0    0     0 1.1039367 0.7171843  0.8967833 0.00000000     0
[14,] 1.0505219    0    0     0 1.2065460 0.7816167  0.8215861 0.00000000     0
[15,] 0.9992439    0    0     0 1.1461330 0.8627980  0.8264179 0.00000000     0
[16,] 0.9378533    0    0     0 1.0823152 0.8996760  0.9021263 0.00000000     0
[17,] 0.9253882    0    0     0 1.0763577 0.9555631  0.9119019 0.00000000     0
[18,] 0.9749932    0    0     0 1.1193464 0.9188745  0.8961368 0.00000000     0
[19,] 0.0000000    0    0     0 0.0000000 0.0000000 -4.7532185 0.00000000     0
[20,] 0.0000000    0    0     0 0.0000000 0.0000000  2.4114109 0.00000000     0
[21,] 0.0000000    0    0     0 0.0000000 0.0000000  0.0000000 0.00000000     0
[22,] 0.0000000    0    0     0 0.0000000 0.0000000  0.0000000 0.00000000     0
[23,] 0.0000000    0    0     0 0.0000000 0.0000000  0.0000000 0.00000000     0
          [,16]      [,17]        [,18] [,19]       [,20]     [,21]       [,22]
 [1,]        NA         NA          NA    NA          NA        NA          NA
 [2,] 0.7736798  0.7270726 0.009925244     0 0.007873429 1.1613597  0.2968975
 [3,] 0.8187679  0.9185148 0.000000000     0 0.000000000 1.2264404  0.3475133
 [4,] 0.7038921  1.1516455 0.137533168     0 0.000000000 0.9516922  0.6551207
 [5,] 0.7038921  1.1516455 0.137533168     0 0.000000000 0.9516922  0.6551207
 [6,] 0.7038921  1.1516455 0.000000000     0 0.000000000 0.9516922  0.6551207
 [7,] 0.7038921  1.1516455 0.000000000     0 0.000000000 0.9516922  0.6551207
 [8,] 0.7038921  1.1516455 0.000000000     0 0.000000000 0.9516922  0.6551207
 [9,] 0.7038921  1.1516455 0.137533168     0 0.000000000 0.9516922  0.6551207
[10,] 0.7038921  1.1516455 0.137533168     0 0.000000000 0.9516922  0.6551207
[11,] 0.7038921  1.1516455 0.000000000     0 0.000000000 0.9516922  0.6551207
[12,] 0.7038921  1.1516455 0.000000000     0 0.000000000 0.9516922  0.6551207
[13,] 0.7038921  1.1516455 0.000000000     0 0.000000000 0.9516922  0.6551207
[14,] 0.7104111  1.1961840 0.148212755     0 0.000000000 1.0830077  0.7077173
[15,] 0.8538310  1.1733537 0.125883507     0 0.000000000 1.0656465  0.7497333
[16,] 0.9015734  1.2531053 0.000000000     0 0.000000000 1.0599166  0.8212694
[17,] 0.9211373  1.2386700 0.000000000     0 0.000000000 1.0230796  0.8415179
[18,] 0.9077512  1.2238167 0.000000000     0 0.000000000 1.0383186  0.8161802
[19,] 0.0000000 12.5800664 0.000000000     0 1.521140638 6.7553059 -4.5837182
[20,] 0.0000000  0.0000000 0.000000000     0 0.000000000 1.9050318  0.0000000
[21,] 0.0000000  0.0000000 0.000000000     0 0.000000000 1.8307783  0.0000000
[22,] 0.0000000  0.0000000 0.000000000     0 0.000000000 1.7237791  0.0000000
[23,] 0.0000000  0.0000000 0.000000000     0 0.000000000 0.0000000  0.0000000
          [,23] [,24]        [,25]     [,26]     [,27]    [,28]     [,29] [,30]
 [1,]        NA    NA           NA        NA        NA       NA        NA    NA
 [2,] 0.5290071     0 3.648741e-05 0.6134642 0.7635759 0.000000  0.000000     0
 [3,] 0.9165404     0 0.000000e+00 0.7510399 0.9622667 0.000000  0.000000     0
 [4,] 0.9789061     0 0.000000e+00 0.9857193 0.7797744 0.000000  0.000000     0
 [5,] 0.9789061     0 0.000000e+00 0.9857193 0.7797744 0.000000  0.000000     0
 [6,] 0.9789061     0 0.000000e+00 0.9857193 0.7797744 0.000000  0.000000     0
 [7,] 0.9789061     0 0.000000e+00 0.9857193 0.7797744 0.000000  0.000000     0
 [8,] 0.9789061     0 0.000000e+00 0.9857193 0.7797744 0.000000  0.000000     0
 [9,] 0.9789061     0 0.000000e+00 0.9857193 0.7797744 0.000000  0.000000     0
[10,] 0.9789061     0 0.000000e+00 0.9857193 0.7797744 0.000000  0.000000     0
[11,] 0.9789061     0 0.000000e+00 0.9857193 0.7797744 0.000000  0.000000     0
[12,] 0.9789061     0 0.000000e+00 0.9857193 0.7797744 0.000000  0.000000     0
[13,] 0.9789061     0 0.000000e+00 0.9857193 0.7797744 0.000000  0.000000     0
```

```
[14,] 1.1895090     0 0.000000e+00 1.1493480 0.8739938 0.000000  0.000000     0
[15,] 1.1183375     0 0.000000e+00 1.1114928 0.9221033 0.000000  0.000000     0
[16,] 1.0547245     0 0.000000e+00 1.1621535 0.9028635 0.000000  0.000000     0
[17,] 1.0493486     0 0.000000e+00 1.1607845 0.8985716 0.000000  0.000000     0
[18,] 1.0816517     0 0.000000e+00 1.2059954 0.9075372 0.000000  0.000000     0
[19,] 0.0000000     0 0.000000e+00 3.0489505 0.0000000 5.742127 -5.857591     0
[20,] 0.0000000     0 0.000000e+00 0.0000000 0.0000000 0.000000  0.000000     0
[21,] 0.0000000     0 0.000000e+00 0.0000000 0.0000000 0.000000  0.000000     0
[22,] 0.0000000     0 0.000000e+00 0.0000000 0.0000000 0.000000  0.000000     0
[23,] 0.0000000     0 0.000000e+00 0.0000000 0.0000000 0.000000  0.000000     0
          [,31]     [,32]      [,33]      [,34]      [,35]     [,36]      [,37]
 [1,]        NA        NA         NA         NA         NA        NA         NA
 [2,] 0.8183553 0.5208135  0.5683264 0.25346349  0.0000000 0.5847074  0.9424044
 [3,] 1.0807633 0.6726109  0.8083805 0.08878318  0.0000000 0.7818306  0.9482598
 [4,] 0.8241077 0.8669748  1.0318836 0.00000000  0.0000000 0.7627749  0.9988159
 [5,] 0.8241077 0.8669748  1.0318836 0.00000000  0.0000000 0.7627749  0.9988159
 [6,] 0.8241077 0.8669748  1.0318836 0.00000000  0.0000000 0.7627749  0.9988159
 [7,] 0.8241077 0.8669748  1.0318836 0.00000000  0.0000000 0.7627749  0.9988159
 [8,] 0.8241077 0.8669748  1.0318836 0.00000000  0.0000000 0.7627749  0.9988159
 [9,] 0.8241077 0.8669748  1.0318836 0.00000000  0.0000000 0.7627749  0.9988159
[10,] 0.8241077 0.8669748  1.0318836 0.00000000  0.0000000 0.7627749  0.9988159
[11,] 0.8241077 0.8669748  1.0318836 0.00000000  0.0000000 0.7627749  0.9988159
[12,] 0.8241077 0.8669748  1.0318836 0.00000000  0.0000000 0.7627749  0.9988159
[13,] 0.8241077 0.8669748  1.0318836 0.00000000  0.0000000 0.7627749  0.9988159
[14,] 0.8764779 1.0213439  1.0703056 0.00000000  0.0000000 0.9378703  1.0378086
[15,] 1.0172809 0.9097299  1.1068804 0.00000000  0.0000000 0.9366468  1.0650998
[16,] 1.1078000 0.9366599  1.0770665 0.00000000  0.0000000 0.9363373  1.0643124
[17,] 1.1234195 0.9121553  1.1295226 0.00000000  0.0000000 0.9274211  1.0612332
[18,] 1.1077831 0.9360675  1.1499083 0.00000000  0.0000000 0.9560186  1.1382328
[19,] 0.0000000 3.0522495 -2.7588208 0.00000000 -9.5882132 1.9078683 -8.0590989
[20,] 0.0000000 1.6679710  0.0000000 0.00000000  0.4031304 0.9679535  1.0181301
[21,] 0.0000000 1.6101832  0.0000000 0.00000000  0.2397078 0.8207908  1.6581060
[22,] 0.0000000 0.0000000  0.0000000 0.00000000  0.5898609 0.4933837  1.8269072
[23,] 0.0000000 0.0000000  0.0000000 0.00000000  0.4568076 0.5860272  1.6344947
           [,38]      [,39]    [,40]
 [1,]         NA         NA       NA
 [2,] 0.00000000  0.0000000 0.000000
 [3,] 0.00000000  0.0000000 0.000000
 [4,] 0.08848129  0.0000000 0.000000
 [5,] 0.08848129  0.0000000 0.000000
 [6,] 0.08848129  0.0000000 0.000000
 [7,] 0.08848129  0.0000000 0.000000
 [8,] 0.00000000  0.0000000 0.000000
 [9,] 0.08848129  0.0000000 0.000000
[10,] 0.08848129  0.0000000 0.000000
[11,] 0.08848129  0.0000000 0.000000
[12,] 0.08848129  0.0000000 0.000000
[13,] 0.00000000  0.0000000 0.000000
[14,] 0.05646344  0.0000000 0.000000
[15,] 0.12129044  0.0000000 0.000000
```

```
[16,]  0.21975358  0.0000000 0.000000
[17,]  0.24170387  0.0000000 0.000000
[18,]  0.00000000  0.0000000 0.000000
[19,]  7.03613406  2.0063138 7.163193
[20,]  0.35137559 -0.2661917 0.000000
[21,] -0.13978472  0.0000000 0.000000
[22,] -0.05838185  0.0000000 0.000000
[23,]  0.08559199  0.0000000 0.000000
```

```
> h_output$S1.PROB[1:40]
```

```
 [1] 0.8 1.0 0.1 0.4 1.0 1.0 0.1 0.2 0.2 1.0 1.0 1.0 0.0 0.1 1.0 1.0 0.7 0.1 0.3
[20] 1.0 1.0 1.0 0.5 0.3 1.0 1.0 0.3 0.1 0.3 1.0 1.0 1.0 0.5 0.2 1.0 1.0 0.9 0.1
[39] 0.1 1.0
```

```
> h_output$S2.PROB[1:40]
```

```
 [1] 1.0 1.0 1.0 1.0 0.7 0.5 0.3 0.5 0.5 0.4 0.3 0.7 0.4 0.4 0.4 0.6 0.5 0.3 0.6
[20] 0.9 0.6 0.2 0.5 0.2 0.6 0.5 0.6 0.6 0.3 0.3 0.8 0.6 0.4 1.0 1.0 1.0 1.0 0.7
[39] 0.6 0.5
```

3 References

Long, Q., and Johnson, B. A. (2015). Variable selection in the presence of missing data: resampling and imputation. Biostatistics, 16(3), 596-610.