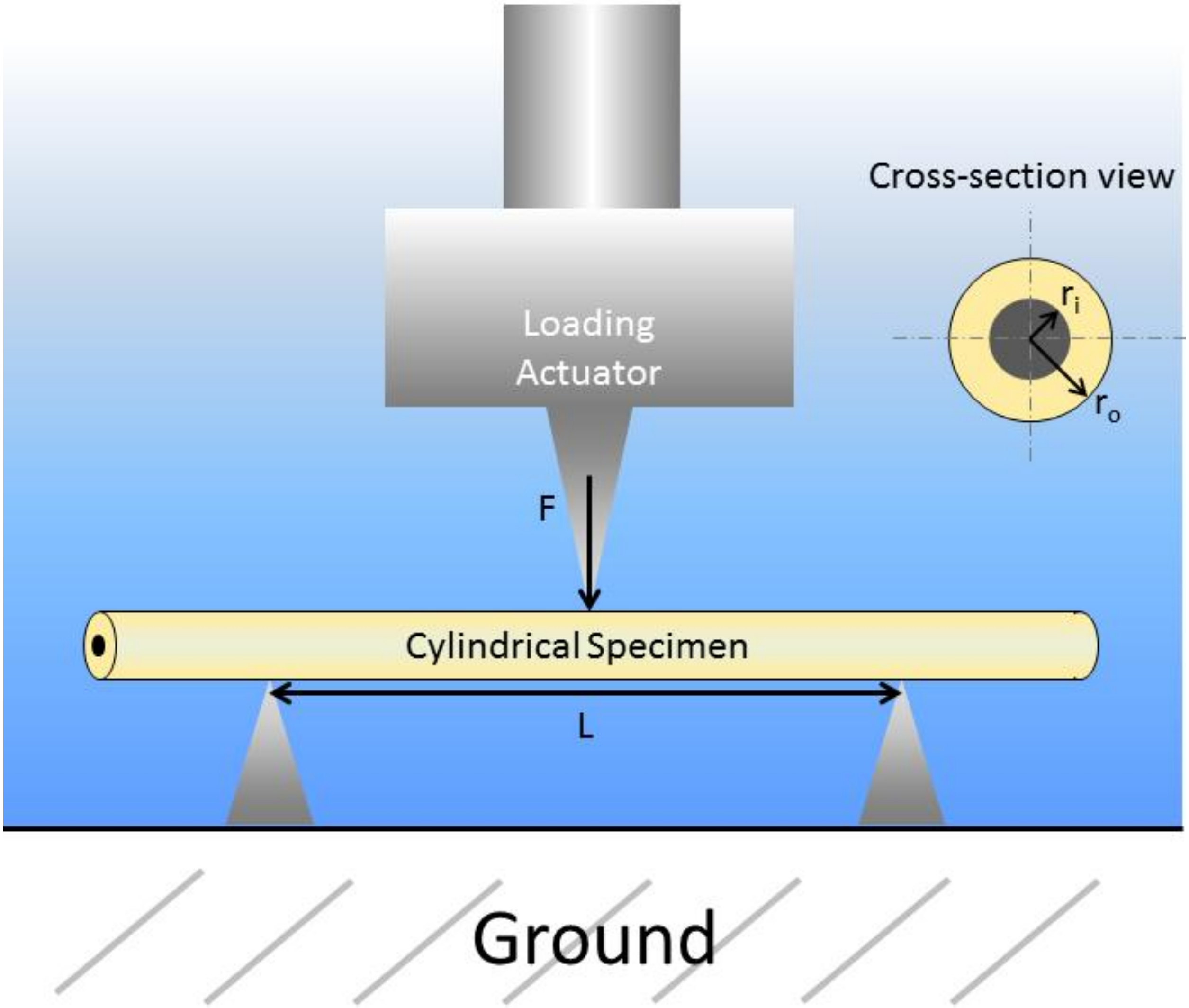
Example 2: Cylindrical Three Point Bending



A cylindrical specimen is subjected to a three-point bending test. The specimen is 10 centimeters long, has an inner radius (r_i) of 2.5 mm and an outer radius (r_o) of 5.0 mm. The specimen is placed on two supports that are 5 cm apart (L), and the actuator is applying a force in the exact middle of the two supports (L/2). Immediately before failure, the Instron records a force (F) of 50N, and a deformation of 2mm. We need to determine the maximum flexural strength (σ) and Young's Modulus (E) of the specimen.

To accomplish this task, we are going to use the two following equations:

$$\sigma = \frac{My}{I} \quad and \quad \delta = \frac{FL^3}{48EI} \qquad \text{(Eq. 2.1 and 2.2)}$$

Where M is the moment (or torque) applied at the middle of the specimen, y is the distance from the center of the specimen to the convex surface, and I is the "polar moment of inertia," a term used to define how the geometry of the specimen influences its reaction to loads.

First, we must calculate the reaction forces at the supports. We have two unknown values, and therefore must use two equations to solve the system. Based on static mechanics, we can use the following two equations:

$$\sum F_y = 0 \quad \text{(Eq. 2.3)}$$

and

$$\sum M_{@Any\ Point} = 0 \quad \text{(Eq. 2.4)}$$

In our case, these equations are as follows:

$$\sum F_y = -F + F_{Left\ Support} + F_{Right\ Support} = 0$$

$$\sum F_y = -50 + F_{Left\ Support} + F_{Right\ Support} = 0$$

Or

$$F_{Left\ Support} + F_{Right\ Support} = 50 \quad \text{(Eq. 2.5)}$$

Using the (Eq. 2.4), we find:

$$\sum M_{@Left\ Support} = \left(-F * \frac{L}{2}\right) + \left(F_{Right\ Support} * L\right) = 0$$

$$\left(-50 * \frac{0.05}{2}\right) + \left(F_{Right\ Support} * 0.05\right) = 0$$

Solving for $F_{Right\ Support}$ we find:

$$F_{Right\ Support} = \frac{\left(50 * \frac{0.05}{2}\right)}{0.05} = 25N$$

Substituting the value of 25N for $F_{Right\ Support}$ back into (Eq. 2.5), we find:

$$F_{Left\ Support} + 25 = 50$$

Therefore

$$F_{Left\ Support} = 25N$$

Now that we have solved for the reaction forces at the supports, we can calculate the moment *acting at the midpoint* of the specimen by looking at half of the specimen and using the following equation:

$$Moment = \sum Force \cdot distance \qquad \text{(Eq. 2.6)}$$

…in our case…

$$M = F_{Right\ Support} \cdot \frac{L}{2}$$

$$M = 25 * 0.025 = 0.625Nm$$

Next we calculate $y,$ the distance from the center of the specimen to the convex surface:

$$y = \frac{h}{2} \qquad \text{(Eq. 2.7)}$$

…in our case…

$$y = \frac{0.010}{2} = 0.005m$$

Finally, we must calculate I*, the* polar moment of inertia, for our rectangular cross-section. The equation for a solid rectangular cross section is:

$$I_{cylinder} = \frac{\pi}{4}({r_o}^4 - {r_i}^4) \qquad \text{(Eq. 2.8)}$$

If we plug in our values, we get:

$$I_{cylinder} = \frac{\pi}{4}([0.005]^4 - [0.0025]^4) = 3.06e^{-7}$$

Now that we have calculated M, y, and I, we are ready calculate the maximum flexural stress of the specimen, by using (Eq. 2.1):

$$\sigma = \frac{My}{I} = \frac{0.625 * 0.005}{3.06e^{-7}} = 1.02e^4 Pa\ or\ 0.0102\ MPa$$

Finally, we can calculate the Young's modulus of the material by plugging in our values to (Eq. 2.2).

$$\delta = \frac{FL^3}{48EI} \ldots or\ by\ rearranging\ E = \frac{FL^3}{48\delta I}$$

Plugging in our values, we get:

$$E = \frac{50 * 0.05^3}{48 * 0.002 * 3.06e^{-7}} = 2.12e^5\ or\ 0.212\ MPa$$