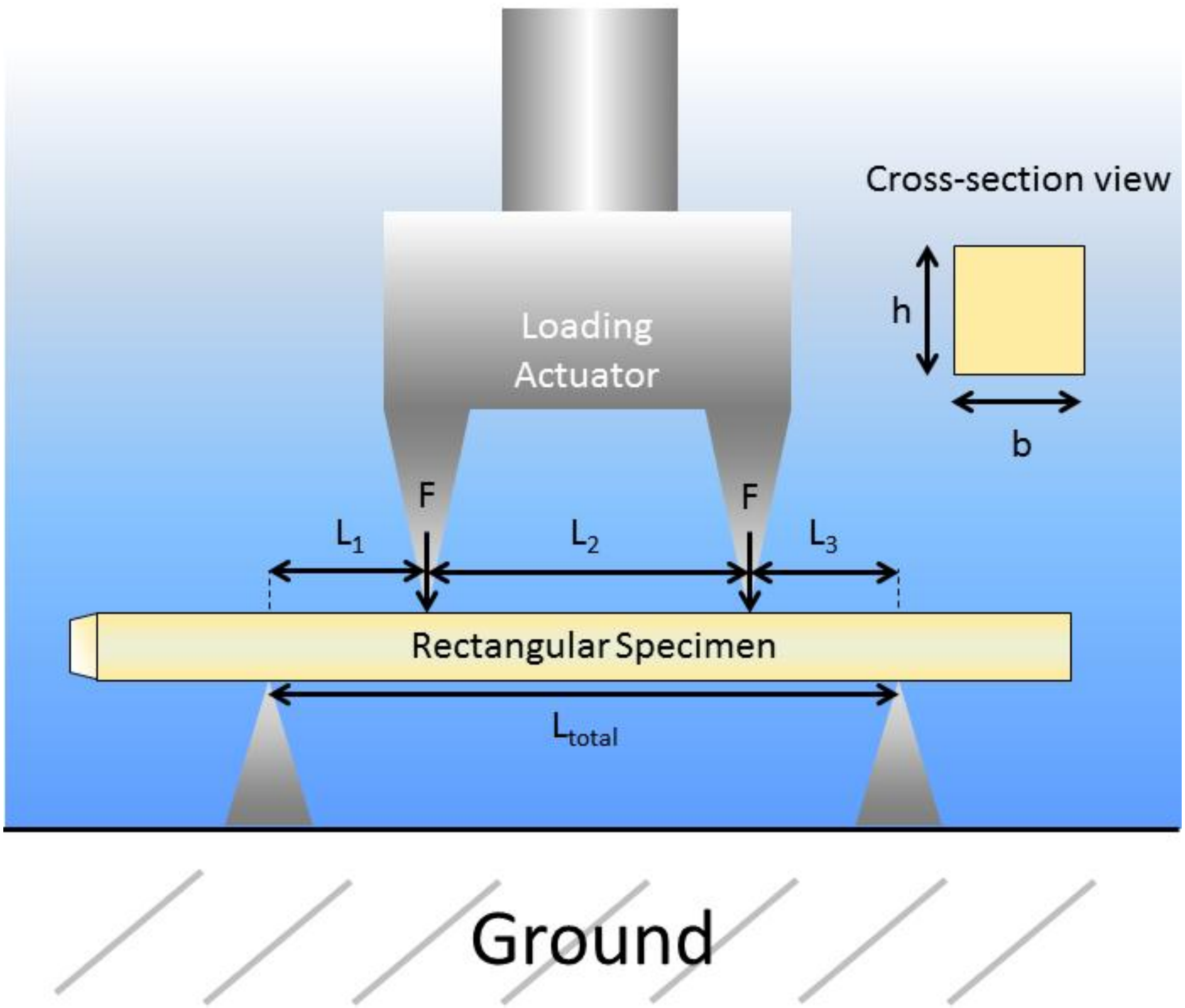
Example 3: Cylindrical Four-point Bending



A rectangluar specimen is subjected to a four-point bending test. The specimen is 10 centimeters long, 10 millimeters wide (b) and 10 millimeters tall (h). The specimen is placed on two supports that are 5 cm apart (L), and the actuator applies two forces on the specimen, 3 cm apart. The distance between the supports and actuator pin is 1 cm on each side. Immediately before failure, the Instron records a force of 50N, and a deformation of 2mm. It can be assumed that the two forces acting upon the specimen (F) are both 25N. We need to determine the maximum flexural strength (σ) and Young's Modulus (E) of the specimen.

For this example, we are going to use the following two equations:

$$\sigma = \frac{My}{I} \quad and \quad \delta = \frac{FL_1(3L_{total}{}^2 - 4L_1{}^2)}{24EI} \qquad \text{(Eq. 3.1 and 3.2)}$$

Where M is the moment (or torque) applied at the middle of the specimen, y is the distance from the center of the specimen to the convex surface (point B in Figure), and I is the "polar moment of inertia," a term used to define how the geometry of the specimen influences its reaction to loads.

First, we must calculate the reaction forces at the supports. We have two unknown values, and therefore must use two equations to solve the system. Based on static mechanics, we can use the following two equations:

$$\sum F_y = 0 \qquad \text{(Eq. 3.3)}$$

and

$$\sum M_{@Any\ Point} = 0 \qquad \text{(Eq. 3.4)}$$

In our case, these equations are as follows:

$$\sum F_y = -2F + F_{Left\ Support} + F_{Right\ Support} = 0$$

$$-50 + F_{Left\ Support} + F_{Right\ Support} = 0$$

Or

$$F_{Left\ Support} + F_{Right\ Support} = 50 \qquad \text{(Eq. 3.5)}$$

Using (Eq. 3.4), we find:

$$\sum M_{@Left\ Support} = (-F * L_1) + (-F * [L_1 + L_2]) + \left(F_{Right\ Support} * [L_1 + L_2 + L_3]\right) = 0$$

$$(-25 * 0.01) + (-25 * 0.04) + \left(F_{Right\ Support} * 0.05\right) = 0$$

Solving for $F_{Right\ Support}$ we find:

$$F_{Right\ Support} = \frac{(25 * 0.01) + (25 * 0.04)}{0.05} = 25N$$

Substituting $F_{Right\ Support}$ back into (Eq. 4.5), we find:

$$F_{Left\ Support} + 25 = 50$$

Therefore

$$F_{Left\ Support} = 25N$$

Now that we have solved for the reaction forces at the supports, we can calculate the moment acting on the element *at the midpoint of the specimen* by looking at half of the specimen and using the following equation:

$$Moment = \sum Force \cdot distance \qquad \text{(Eq. 3.6)}$$

…in our case…

$$M = \left(-F_{Right\ Actuator} * L_{middle\ to\ Right\ Actuator}\right) + \left(F_{Right\ Support} * L_{middle\ to\ Right\ Support}\right)$$

$$M = (-25 * 0.015) + (25 * 0.025) = 0.25Nm$$

Next we calculate y, the distance from the center of the specimen to the convex surface:

$$y = r_o \qquad \text{(Eq. 3.7)}$$

…in our case…

$$y = 0.005\ m$$

Finally, we must calculate I, *the* polar moment of inertia, for our cylindrical cross-section. The equation for a cylindrical cross section is:

$$I_{rectangle} = \frac{bh^3}{12} \qquad \text{(Eq. 3.8)}$$

If we plug in our values, we get:

$$I_{rectangle} = \frac{0.01 * 0.01^3}{12} = 8.33e^{-10}$$

Now that we have calculated M, y, and I, we are ready calculate the maximum flexural stress of the specimen, by using (Eq. 3.1):

$$\sigma = \frac{My}{I} = \frac{0.25 * 0.005}{8.33e^{-10}} = 1.50e^6 Pa\ or\ 1.50\ MPa$$

Finally, we can calculate the Young's modulus of the material by plugging in our values to (Eq. 4.2).

$$\delta = \frac{FL_1\left(3L_{total}{}^2 - 4L_1{}^2\right)}{24EI} \ldots or\ by\ rearranging\ \ E = \frac{FL_1\left(3L_{total}{}^2 - 4L_1{}^2\right)}{24\delta I}$$

Plugging in our values, we get:

$$E = \frac{50 * 0.01(3 * 0.05^2 - 4 * 0.01^2)}{24 * 0.002 * 8.33e^{-10}} = 8.87e^7\ or\ 88.7\ MPa$$